

Waves on Strings

Traveling Waves Giving Rise To Standing Wave Patterns on Thin Strings

January 20, 2017

You will **not return** this handout to the instructor at the end of the lab period.

The Waves on Strings lab has two parts.

This lab consists of **Activities #1, #2, #3, and #4**.

Work through the activities in the usual manner in the lab, and hand in your results at the end of the regularly scheduled lab period.

Instructions

Before the lab, read all sections of the *Introduction to Waves on Stretched Strings*, and answer the Pre-Lab questions on the last page of this handout. Hand in your answers as you enter the general physics lab.

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Waves on Strings

0. Introduction to waves on stretched springs

References James S. Walker, *Physics*, 4th edition, chapter 14, especially section 14.8
 Randall D. Knight, *Physics for Scientists and Engineers*, 3rd edition, chapter 21, especially section 21.3

0.1 A traveling wave on a stretched string

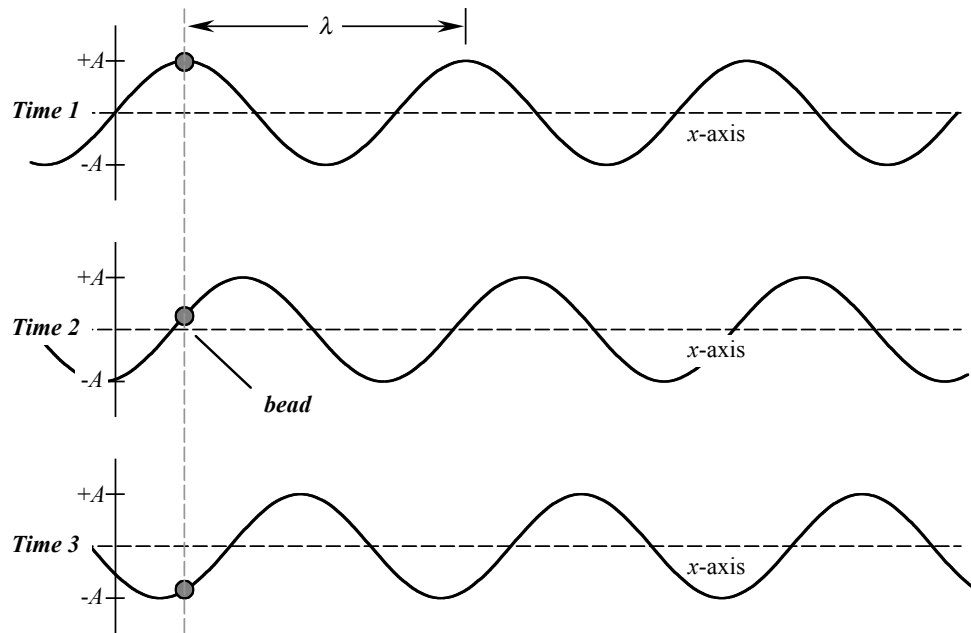


Figure 1 This figure shows a series of snapshots of a traveling wave on a long stretched string. The up and down vibration moves from left to right even though the string itself has no velocity in the $+x$ direction. Note the bead – glued to the string – which just rises and falls as the wave moves left to right. The dashed vertical line is to indicate that the x -position of the bead never changes as it moves up and down.

Look at Figure 1. The solid line represents a thin string. Originally, the string was stretched tightly and lay in the position indicated by the dotted line, but someone plucked or bowed the string, causing it to vibrate. The vibration travels along the length of the string. Figure 1 shows the vibration traveling from left to right, but vibrations can also travel from right to left.

If you think of ripples moving along the surface of a puddle or a lake, it may help you to visualize the vibration that travels along the string. The up-and-down vibration of ripples moving along the surface of the water is very much like the up-and-down vibration that moves along the length of the string. Figure 1 is like a series of snapshots, showing the string at successive instants of time. A movie of the string would show the vibration sliding along the string from left to right.

Figure 1 shows the simplest kind of a wave. A wave in air – a sound wave – with the same form of vibration is heard as a pure musical tone, a fundamental with no overtones. The waves with overtones have more complicated shapes.

0.2 Describing a wave on a stretched string

All waves, including waves on stretched strings, are characterized by the following.

Wave velocity If you watch one peak, it moves from left to right. The speed with which the peak moves is the velocity of the wave, v , in meters per second.

Wavelength The distance between two adjacent peaks is the wavelength, λ , in meters. The letter λ is a Greek lowercase lambda.

Frequency The string in Figure 1 has a small bead glued to it. The bead moves up and down in simple harmonic motion. (The vibration of the string appears to move left to right, but the string is not going anywhere; parts of the string just move up-and-down, and the ups and downs move left to right along a string which has no sideways motion.) The number of times in one second the bead goes from up to down and back to up is the frequency, f . Frequency is the number of complete vibrations of a point on the string in one second, that is, vibrations per second. One vibration per second is called one hertz, so frequency is f hertz.

Amplitude Take the horizontal straight dotted line in Figure 1 to be an x -axis. Then the bead moves up and down between a maximum height A above the x -axis and a maximum depth $-A$ below the x -axis. The distance A is the amplitude of the wave, in meters.

Period The time it takes the bead to go from up to down and back to up is called the period, T , in seconds. T is also the time it takes for a peak in the wave to move distance λ . This is obvious if you visualize what the string is doing.

0.3 Relations between v , λ , f , and T

0.3.1 The number of vibrations per second multiplied by the number of seconds for one vibration equals one vibration. (That is supposed to be obvious.) Hence

$$fT = 1.$$

0.3.2 The speed of the wave is the distance a peak moves divided by the time it takes to move that distance. Since the peak moves distance λ in time T , $v = \lambda/T$. From $fT = 1$, it follows that $f = 1/T$, so the velocity can be written

$$v = f\lambda.$$

0.4 The speed of a wave depends on the string

Let σ represent the mass of a piece of string that is one meter long, in kilograms per meter. The letter σ is a Greek lowercase sigma. σ is called the *linear mass density* of the string. For any string, $\sigma = M/L$, where M is the mass of the string (in kilograms) and L is the length of the string (in meters).

Let F be the tension within the stretched spring, a force, measured in newtons.

The following formula shows how to calculate the speed of the wave if you know the string's tension F and linear mass density σ .

$$v = \sqrt{\frac{F}{\sigma}}$$

0.5 The shape of the wave on the stretched string

Continue to take the horizontal straight dotted line in Figure 1 as the x -axis. Add a y -axis in order to be able to keep track of how high above the x -axis a piece of the string is. Note the y -axis in Figure 1.

A wave traveling in the $+x$ direction with amplitude A , frequency f , and wavelength λ has the same shape as the graph of the following equation.*

$$y = A \cos\left(2\pi ft - \frac{2\pi x}{\lambda} + \varphi\right)$$

If you know the amplitude A , the frequency f , the wavelength λ , and the phase factor φ , you can use this equation to calculate the height y of the piece of string at position x when the time is t . The use of this equation is to tell what every part of the string is doing at all times.

If the wave is traveling in the $-x$ direction, the equation is the same except that the minus sign changes to a plus sign.

$$y = A \cos\left(2\pi ft + \frac{2\pi x}{\lambda} + \varphi\right)$$

If you are assigned to do the computer simulation in Activity #5, you will see the motion of waves on a string using these equations.

0.6 Waves on guitar, violin, and piano strings



Figure 2 This figure shows a string held firmly at both ends. The small wave on the left moves to the right and gets turned up side down when it reflects off the right end. If the waves are longer, covering the whole string, then everywhere on the string there are two waves, one up side down compared to the other.

Guitar, violin, and piano strings are all held firmly in place at both ends. You start guitar strings vibrating by plucking. Violin strings are bowed. Piano strings are hit with a hammer.

Whatever the method, it causes many waves of many different frequencies to travel from one end of the string to the other. When the waves arrive at the other end of the string, they reflect by being turned upside down and sent back in the other direction. Reflecting repeats at the other end of the string and continues indefinitely as waves travel back and forth along the string, bouncing off each end.

If one wave tries to make the string go up (in the $+y$ direction) while another wave tries to make the string go down ($-y$ direction), the opposing efforts cancel each other out, so nothing happens. This has a name: *destructive interference*, because the waves interfere with each other in a way that is “destructive.” However, it is possible to have a pair of waves with the same amplitude, frequency, and phase but traveling in opposite directions. Because they have the same phase, they both move the string in the $+y$ direction at the same time, and they both move

* Some people use sine instead of cosine. Both sine and cosine have the same shape, so one can use either to describe pure tones.

the string in the $-y$ direction at the same time. Instead of canceling each other out, they help each other. The name for this is *constructive interference*.

0.7 The shapes of waves on guitar, violin, and piano strings

The following discussion remains restricted to pure tones. More complicated shapes are combinations of a fundamental pure tone plus overtones.

Because of the reflections from the ends of the string, and because only waves of the same frequency and phase do not cancel each other, a string vibrating as a pure tone always holds two waves of the same frequency and amplitude but traveling in opposite directions, with one wave upside down compared to the other. The equation that describes the shape of the string is the following.

$$y = \left[+ A \cos\left(2\pi ft - \frac{2\pi x}{\lambda} + \varphi\right) \right] + \left[- A \cos\left(2\pi ft + \frac{2\pi x}{\lambda} + \varphi\right) \right]$$

The term with the amplitude $+A$ and the minus inside the $\cos(\dots)$ is the wave traveling left to right. The term with amplitude $-A$ and all plus signs inside the cosine is the upside down wave traveling right to left. By adding the two terms, you get the shape of the string when both waves are present simultaneously. This is an example of the *principle of superposition*, according to which you can superimpose two different waves simply by adding their individual formulae.

Because of the trigonometric identity

$$\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2 \sin(\alpha) \sin(\beta),$$

the above expression for y is the same as the next, simpler expression.

$$y = 2A \sin(2\pi ft + \varphi) \sin\left(\frac{2\pi x}{\lambda}\right)$$

You will see this string shape in your laboratory work and also if you are assigned to do the computer simulation in Activity #5. Figure 3 is an example of what these shapes look like.

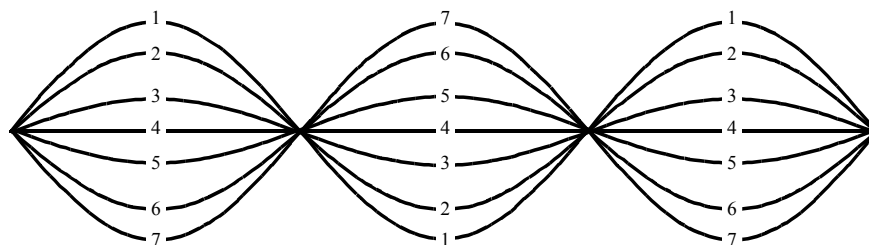


Figure 3 The string vibration shown here is called a standing wave because the peaks just move up-and-down (in the $\pm y$ direction) without moving left or right ($\pm x$ direction). This figure shows a series of seven snapshots of the string. At the time 1, the string has the shape of curve 1. A short time later, at time 2, the string has the shape of curve 2. And so forth. The sequence of wave shapes $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 6 \rightarrow 5 \rightarrow \dots$ repeats indefinitely.

In Figure 3, the string is shown divided into three vibrating regions separated by two *nodes* (a node is a point which is motionless even though the rest of the string around it is in motion). The number of vibrating regions in a string need not be 3; it can be 1, or 2, or 3, or See Figure 4, which shows strings of length L divided into 1, 2, and 3 vibrating regions.

Waves on Strings

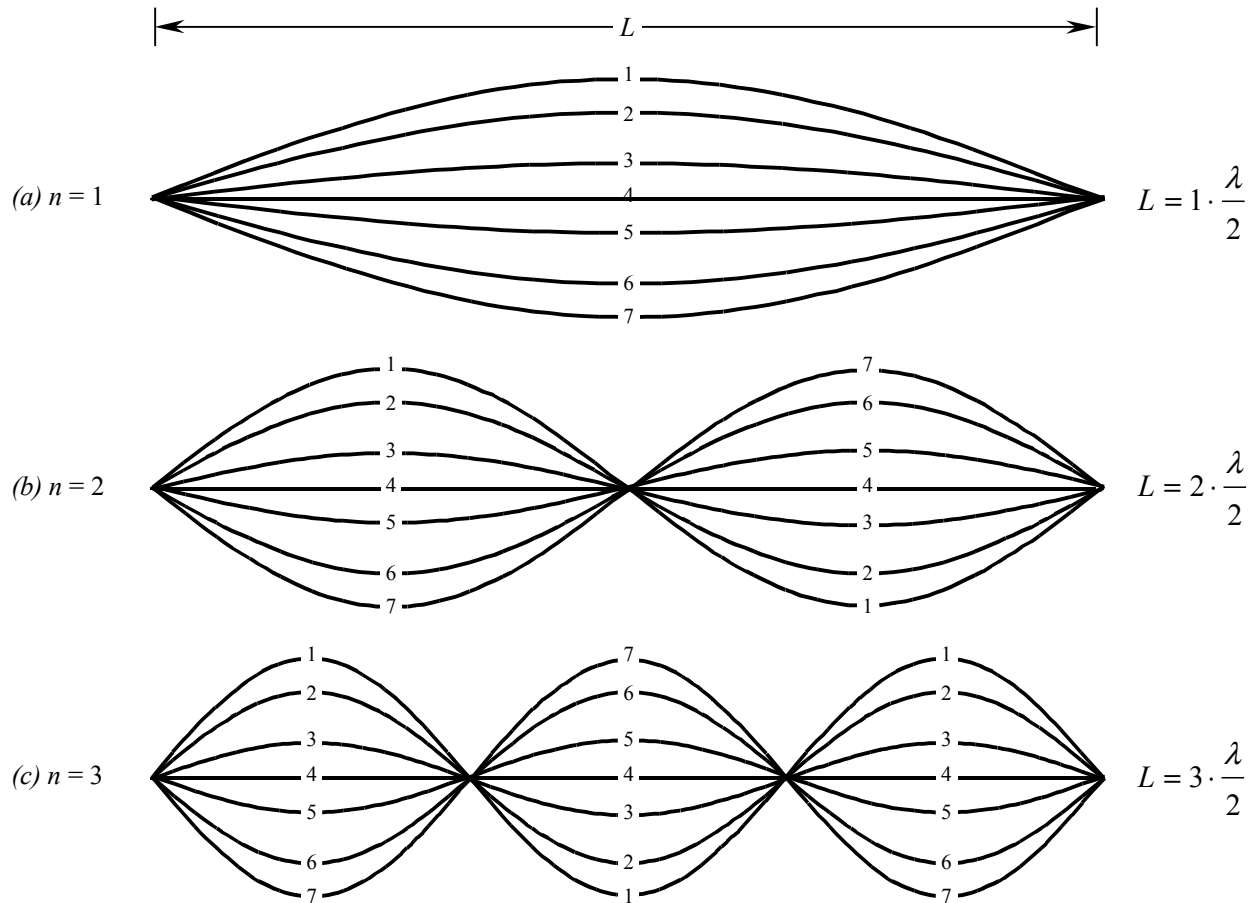


Figure 4 Note the relation between wavelength and string length: $L = n \frac{\lambda}{2}$, where n is the number of vibrating regions in the string of length L .

As is evident from Figure 4, if n is the number of vibrating regions of the string, the relation between the length of the string L and the wavelength λ is $L = n \cdot (\lambda/2)$. Solving this for the wavelength λ , you obtain a relation that says if the string length is L , the wavelength must be

$$\lambda = \frac{2L}{n}, \text{ where } n \text{ is an integer: } 1, 2, 3, \dots$$

In other words, on a fixed length string pinned at both ends, a vibration cannot have just any wavelength; it can only have one of the wavelengths related to the string length by $\lambda = 2L/n$, where n is one of 1, 2, 3,

0.8 How stringed musical instruments produce different tones

Beginning with the three relations

$$v = f\lambda, \quad v = \sqrt{\frac{F}{\sigma}}, \text{ and} \quad \lambda = \frac{2L}{n},$$

algebraically eliminate v and λ , and then solve for f . The result is

$$f = \frac{n}{2L} \sqrt{\frac{F}{\sigma}}$$

It is possible to make a vibrating string have 2, 3, or more vibrating regions, but in normal use of a stringed instrument there is just one vibrating region (see the $n = 1$ part of Figure 4), so normally $n = 1$, and

$$f = \frac{1}{2L} \sqrt{\frac{F}{\sigma}}$$

Using this formula, if you know the tension F , the linear density σ , and the length of the string L , you can calculate the frequency f of the tone the string produces. In violins and guitars, the tension and linear density of a given string are unchanging while the musician changes the length of the string by pressing down with the fingers of the left hand. Changing the length changes the tone. From the formula, small lengths produce larger frequencies and therefore higher pitched tones.

0.9 Today's lab activities

Activity #1 will be a demonstration by your lab instructor. Activity #2 will be a variation of Melde's experiment. Melde's experiment uses a string stretched between two points and made to vibrate by a vibrator attached to one end; see Figure 5.

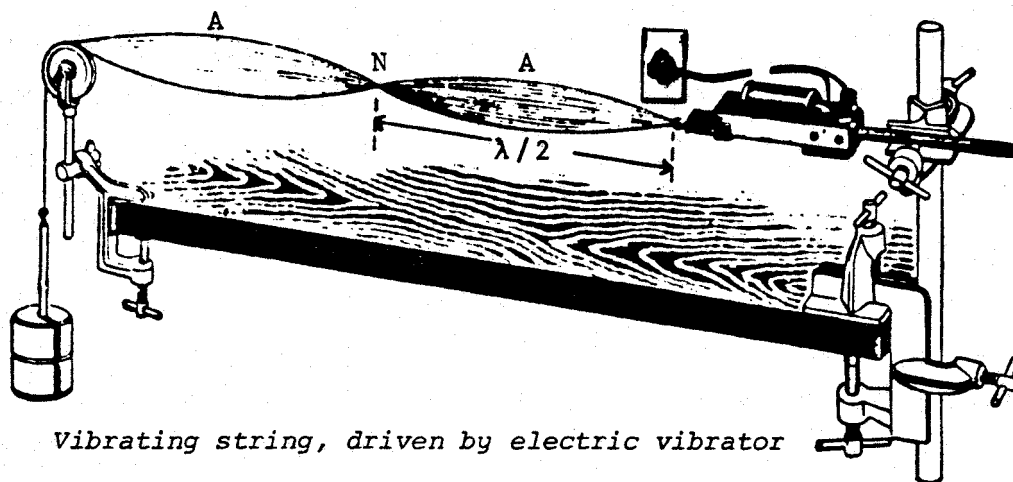


Figure 5 The apparatus for Melde's Experiment. The vibrator frequency, and hence the wave frequency, is 60 vibrations per second (60 Hz).

To see what is to be done in Activity #2, begin with the equation (from section 0.8)

$$f = \frac{n}{2L} \sqrt{\frac{F}{\sigma}}$$

Solve for n , and adjust the form slightly to get the following.

$$n = 2Lf \sqrt{\sigma} \cdot \frac{1}{\sqrt{F}}$$

In Melde's experiment, L (length of string), f (vibrator frequency), and σ (linear mass

density of string) are fixed and cannot be changed. F , the tension in the string, can be varied by adding or removing weight to or from the weight hanger attached to the string.

Knowing L , f , and σ , you can calculate n , the number of vibrating regions in the string, for different values of tension F .

However, if the values of L , f , and σ are such that n is not an integer, the string may tremble, but it will not have a full standing wave vibration shape. That is because when n is a fraction or decimal (like $3/4$, 2.55 , ...), the waves moving in the $+x$ direction and the waves moving in the $-x$ direction are not in phase, and they pretty much cancel each other. Destructive interference occurs.

On the other hand, when the values of L , f , and σ are such that n is an integer, the interference is constructive, and a beautiful standing wave – like those in Figure 4 – appears.

Here then is what you will do in Activity #2. First determine, by trial and error, how much mass m to put on the weight hanger to create a tension F that causes a standing wave with $n = 3$, that is, with three vibrating regions. Then you adjust the mass until you get a standing wave with $n = 4$, that is, with four vibrating regions. Continue on to larger and larger values of n .

Large values of n are hard to obtain, and eventually you will be forced to stop. When you have gone as far as you can, you will have a table of values of n and the corresponding tensions F . The above equation says that if you make a graph with n on the vertical axis and $1/\sqrt{F}$ on the horizontal axis, you will get a straight line passing through the origin with slope equal to $2Lf\sqrt{\sigma}$.

Seeing that this graph is a straight line through the origin and comparing the slope of the line with a directly calculated value of $2Lf\sqrt{\sigma}$ will show you that the theory is correct.

1. Activity #1: Demonstration of standing waves on a string

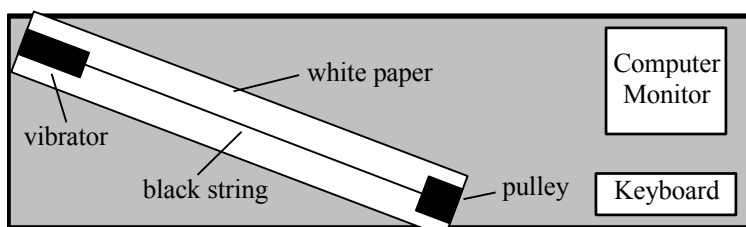


Figure 6 Layout of the work table

Equipment: Electric vibrator, $f = 60$ Hz
 String (30G black wire, a bit less than 2 m long)
 White paper ~10 cm wide by ~2 m long on table, to make the string visible
 Adding machine tape works well.
 Weight hanger (50 g)
 Hanger slotted weight set (in grams: 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500)
 A metric measuring stick two meters long
 Pulley and clamps
 Stroboscope

1.1 The lab instructor shows how to obtain a standing wave with $n = 3$.

1.2 The lab instructor uses a stroboscope to stop the motion of the string and to show the string moving in slow motion.

Do not spend more than about one hour on activities #2 and #3.

2. Activity #2: Standing waves for $n = 3, 4, 5, \dots$

Abstract Locate your own $n = 3$ standing wave, and then locate standing waves for $n = 4, 5, \dots$

Waves on strings			
Date ...			
Time ...			
Partner name #1			
Partner name #2			
Partner name #3			
$g =$	9.8	m/s ²	
$f =$		Hz	
$L =$		m	
$\sigma =$		kg/m	
$2 L f \sigma^{1/2} =$			
slope =			
Discrepancy =		%	
Mass (kg)	$F = \text{Mass} \cdot g$ (newtons)	$1/F^{1/2}$	n
			3
			4
			5
			6
			7
			8
			9
			10
			11
			12
			13
			14
			15
			16
			17
			18
			19
			20

Figure 7 A model for your spreadsheet



- Equipment:** Electric vibrator, $f = 60$ Hz
 String (30G black wire, less than 2 m long)
 string mass density $\sigma = 0.00108$ kg/m
 White paper ~ 10 cm wide by ~ 2 m long on table, to make the string visible
 Adding machine tape works well.
 Weight hanger (50 g)
 Hanger slotted weight set (in grams: 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500)
 A metric measuring stick two meters long
 Pulley and clamps
 PC with MS Excel

2.1 Preliminaries

- 2.1.1 Unless your instructor tells you otherwise, σ , the linear mass density of the string has the value: $\sigma = 0.00108$ kg/meter.
- 2.1.2 Referring to Figure 5 as necessary, tie a piece of the same kind of string to the vibrator, and run the string over the pulley. Make a loop in the end of the string from which to suspend the weight hanger.
- 2.1.3 Referring now to Figure 8, measure $L_{\text{vibrating}}$, the length of the string between the vibrator and the pulley. Measure **from where the string touches the vibrator to where the string touches the pulley.**

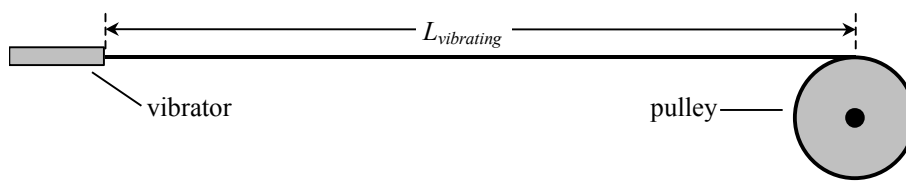


Figure 8 How to measure the string length, $L_{\text{vibrating}}$

- 2.1.4 Turn on the computer, run MS Excel, and set up a spreadsheet modeled after that in Figure 7, above.
- ## 2.2 Standing wave with $n = 3$
- 2.2.1 Turn on the vibrator, and adjust the mass on the weight hanger until you have the best possible standing wave pattern with $n = 3$. **The standing wave pattern is best when it has the largest amplitude.**
- 2.2.2 Record the total mass (added mass plus mass of weight hanger) in the $n = 3$ row of your spreadsheet.
- ## 2.3 Standing waves with $n = 4, 5, 6, \dots$

- 2.3.1 Adjust the mass on the weight hanger for the best $n = 4$ standing wave (largest vibration amplitude) and add the total mass (i.e., including the mass of the weight hanger) to the $n = 4$ row in the spreadsheet. **An extra 1 or 2 g can make a big difference!**
- 2.3.2 Repeat for $n = 5, 6$, etc. Stop when you can no longer tell what mass produces the largest amplitude vibration of the string. You will *not* be able to go all the way to $n = 20$.

2.4 Create the graph

2.4.1 Use spreadsheet formulae to calculate values for the middle two columns of the spreadsheet.

2.4.2 Select the $1/\sqrt{F}$ and n columns, and make a graph (scatterplot) of n versus $1/\sqrt{F}$. Choosing to create the graph as an object in the spreadsheet saves paper.

2.4.3 Add a straight line fit to the graph (right click a data point on the graph and choose **Add Trendline** from the menu). Click the **Options** tab, and

- check the box that makes the **intercept zero**, and also
- check the box that causes MS Excel to display the equation of the linear fit on the graph.

3. Activity #3: The slope of the linear fit

Abstract Compare the slope of the straight line fit with the prediction of the theory.

3.1 You already have on your graph the best straight line fit to the data points and the equation of that line. The equation will be of the form $y = mx$, with no intercept. (If it is not, consult your lab instructor.)

3.2 If the slope m does not show at least three significant figures, change its format so that it does. To do this, right click the equation, and select the **Format Data Labels** option from the menu. Click the **Number** tab. Choose either the **Number** or the **Scientific** option, and adjust the number of digits so that the slope is displayed with at least three significant digits.

3.3 In a new cell, have the spreadsheet calculate the value

$$2L_{\text{vibrating}} f \sqrt{\sigma}.$$

3.4 The value of $2L_{\text{vibrating}} f \sqrt{\sigma}$ calculated by the spreadsheet should be close to the value of the slope of the straight line. To make a comparison, add to the spreadsheet a calculation of the difference between them divided by their average, and express the result as a percent:

$$100 \cdot \frac{(2L_{\text{vibrating}} f \sqrt{\sigma}) - \text{slope}}{\frac{1}{2}[(2L_{\text{vibrating}} f \sqrt{\sigma}) + \text{slope}]}$$

Values exceeding 4% are a cause for concern. One thing you can check are the units; all quantities must be in kilograms, meters, seconds or combinations of those units (e.g., σ must be in kg/m).

3.5 When everything looks correct,

3.5.1 print **one copy of the spreadsheet** for each person in your group, and

3.5.2 print **one copy of the graph** for each person in your group.

Print Your Name

Print Your Partners' Names

4. Activity #4: Questions and what to hand in when you are finished

Abstract Thinking about the lab activities ...

Q 1 In a somewhat different Melde's experiment, imagine that the pulley can be moved back and forth, making $L_{\text{vibrating}}$ greater and smaller while leaving f , σ , and F unchanged. In this experiment, f is 120 Hz, σ is 4.28×10^{-4} kg/m, and F is 6.40 newtons.

(a) When the string vibrates with the maximum amplitude in three segments, what is $L_{\text{vibrating}}$?

Method: Use $n = 2L_{\text{vibrating}} f \sqrt{\sigma} \cdot \frac{1}{\sqrt{F}}$, from page 7. Show your calculation.

(b) When the string vibrates with the maximum amplitude in four segments, what is $L_{\text{vibrating}}$?
Method: Same as (a). Show your calculation.

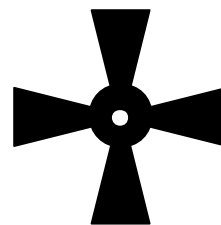
(c) Describe the ways the string would vibrate while the pulley moves so that $L_{\text{vibrating}}$ slowly changes from the value at which $n = 3$ to the value at which $n = 4$ and on past the value at which $n = 4$.

Q 2 *Traveling waves move along stretched strings with constant velocities given by $v = \sqrt{F / \sigma}$. Is it correct to say that all parts of the string have zero acceleration? Explain your answer.*

Q 3 *The lab activities involved standing waves, waves that do not move but just oscillate in place. However, as discussed in section 0, the Introduction, and demonstrated in the computer simulation in Activity #5, these standing waves are the result of two traveling waves, one moving left to right at speed v , and the other identical except for being upside down and traveling right to left. Calculate the speeds of traveling waves on the string you used in lab when $n = 3$ and when $n = 7$. Show your calculations, and give your answers in m/s and in miles/hour (1 m/s = 2.237 miles per hour).*

Q 4 *A fan has four identical, symmetrically placed blades. The blades are rotating clockwise at twenty revolutions per second.*

(a) What is the smallest time interval between stroboscope flashes that will make the fan blades appear motionless? Explain.



(b) What is the highest frequency (in flashes per second) at which a stroboscope will make the fan blades appear to stand still? Show your calculation.

(c) The same questions as (a) and (b), but someone has put a yellow dot on one blade, and now you want the yellow dot to appear to be standing still. Explain, and show your calculation.

(d) Now the stroboscope is set for nineteen flashes per second, and the yellow dot appears to be slowly rotating. Which direction does it appear to rotate, clockwise or counterclockwise? Explain, and show your calculation.

(e) The same as (d), but the stroboscope is set for twenty-one flashes per second. Explain, and show your calculation.

Q 5 *A very long cord hangs from a very high ceiling. Someone wiggles the lower end of the cord, causing a transverse wave to ripple up the cord, moving from the bottom of the cord, near the floor, toward the top of the cord, near the ceiling. As the wave moves upward, does its speed increase, stay the same, or decrease? Justify your answer. Hint Section 0, the Introduction, will be helpful.*

4.1 When you are finished, hand in the following.

4.1.1 The spreadsheet and graph you printed in section 3.5 (one copy of each)

4.1.2 Questions **Q1** through **Q5**, with answers (detach the questions from this handout)

4.2 Do not turn in:

4.2.1 The second copy of the spreadsheet, if you made one.

4.2.2 The rest of the handout (whatever is left after you remove the questions)

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Pre-Lab Questions

Print Your Name

Read the *Introduction* to this handout, and answer the following questions before you come to General Physics Lab. Write your answers directly on this page. When you enter the lab, tear off this page and hand it in.

1. What is *wave velocity*?
2. What is the *wavelength* of a wave?
3. Explain *wave frequency*.
4. Explain what is meant by the *amplitude* of a wave?
5. A wave has amplitude $A = 0.01$ m. What distance does a part of the string travel during one full cycle?
6. Explain the *period* of a wave.
7. Write the formula that expresses the relation between period and frequency.
8. What is the *principle of superposition*?
9. What pattern results when two sinusoidal waves, with one upside down, travel in opposite directions along a stretched string?
10. A piece of string 185 *centimeters* in length has a mass of 2.47 *grams*. What is its linear mass density σ in *kilograms per meter*?
11. What is the frequency of the vibrating string in these lab activities?
12. Describe a standing wave with three loops, including an explanation of how it moves. A drawing is very helpful.

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